Section 2.4 Solutions:

#1-22: Find the derivative of each exponential function

1) $y = e^{3x}$

Rule needed $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$C = 1$

$g(x) = 3x \quad g'(x) = 3$

$$\frac{dy}{dx} = 1 \cdot 3e^{3x}$$

$$\frac{dy}{dx} = 3e^{3x}$$

Answer $\frac{dy}{dx} = 3e^{3x}$

3) $f(x) = e^{4x+5}$

Rule needed $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$c = 1$

$g(x) = 4x+5$

$g'(x) = 4$

$f'(x) = 1 \cdot 4 e^{4x+5}$

$f'(x) = 4e^{4x+5}$

Answer $f'(x) = 4e^{4x+5}$

5) $f(t) = e^{t^2+3t}$

Rule needed $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$C = 1$

$g(T) = T^2 + 3T$

$g'(T) = 2T + 3$

$f'(T) = 1 \cdot (2T+3)e^{T^2+3T}$

$f'(T) = (2T+3)e^{T^2+3T}$

answer: $f'(t) = (2t + 3)e^{t^2+3t}$

7) $f(x) = 2e^{4x}$

Rule needed $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$c = 2$

$g(x) = 4x$

$g'(x) = 4$

$f'(x) = 2 \cdot 4 e^{4x}$

$f'(x) = 8e^{4x}$

answer: $f^{'}(x) = 8e^{4x}$

9) $y = x^2 e^x$

Rule needed for the "e" $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$c = 1 \quad g(x) = x$

$g'(x) = 1$

$1 \cdot 1 \cdot e^x$

Also need the product rule as both factors have an x.

First factor x^2	Second Factor e^x
Derivative $2x$	Derivative e^x
cross multiply top down x^2e^x	*cross multiply bottom up* $2xe^x$

$y' = x^2e^x + 2xe^x$

$y' = x(xe^x + 2e^x)$

answer: $y' = xe^x(x + 2)$

$y' = xe^x(x+2)$

11) $k(y) = (y+2)e^{3y}$

Rule needed for the "e" $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$C = 1 \quad g(y) = 3y$

$g'(y) = 3$

$1 \cdot 3e^{3y}$

Also need the product rule as both factors have an x.

First factor $y+2$	Second Factor e^{3y}
Derivative 1	Derivative $3e^{3y}$
cross multiply top down $(y+2) \cdot 3e^{3y}$	*cross multiply bottom up* $1 \cdot e^{3y}$

$$K'(y) = (y+2) \cdot 3e^{3y} + 1e^{3y}$$

$$K'(y) = e^{3y}\left[(y+2) \cdot 3 + 1\right]$$

~~Answer:~~ $\cancel{y' = xe^{x}(x+2)}$

$$K'(y) = e^{3y}(3y+6+1)$$

$$K'(y) = e^{3y}(3y+7)$$

13) $f(x) = xe^{5x}$

Rule needed for the "e" $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

$c = 1$ $g(x) = 5x$

$g'(x) = 5$

$1 \cdot 5e^{5x}$

Also need the product rule as both factors have an x.

First factor x	Second Factor e^{5x}
Derivative 1	Derivative $5e^{5x}$
cross multiply top down $x \cdot 5e^{5x}$	*cross multiply bottom up* $1e^{5x}$

$$f'(x) = x \cdot 5e^{5x} + 1e^{5x}$$

$$f'(x) = e^{5x}(5x+1)$$

answer: $f'(x) = e^{5x}(5x + 1)$

15) $f(t) = \frac{t^2}{e^t}$

Rule needed for the "e"
$f(x) = ce^{g(x)}$
$f'(x) = cg'(x)e^{g(x)}$
Where "c" is a constant (number without a letter)

Also need the quotient rule because of the division.

Denominator	e^T	Numerator	T^2
Derivative	e^T	Derivative	$2T$
cross multiply top down	$2Te^T$	*cross multiply bottom up*	T^2e^T

$$f'(T) = \frac{2Te^T - T^2e^T}{(e^T)^2}$$

$$f'(T) = \frac{Te^T(2-T)}{e^T \cdot e^T}$$

$$\boxed{f'(T) = \frac{T(2-T)}{e^T}}$$

OKAY TO STOP here

$$f'(t) = \frac{T(-T+2)}{e^T}$$

$$f'(t) = \frac{-T(T-2)}{e^T}$$

answer $f'(t) = \frac{-t^2+2t}{e^t} = \frac{-t(t-2)}{e^t}$

17) $f(x) = \frac{x+2}{e^x}$

Rule needed for the "e" $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the quotient rule because of the division.

Denominator	e^x	Numerator	$x+2$
Derivative	e^x	Derivative	1
cross multiply top down	$1e^x$	*cross multiply bottom up*	$(x+2)e^x$

$$f'(x) = \frac{1e^x - e^x(x+2)}{(e^x)^2}$$

$$f'(x) = \frac{e^x(1-(x+2))}{e^x \cdot e^x}$$

answer $f'(x) = \frac{-1x-1}{e^x} = \frac{-1(x+1)}{e^x}$

$$f'(x) = \frac{1-x-2}{e^x}$$

$$f'(x) = \frac{-x-1}{e^x}$$

19) $f(x) = 3^x$

Rule needed $f'(x) = c\ln(a) g'(x) a^{g(x)}$

$c = 1$

$a = 3$

$g(x) = x$

$g'(x) = 1$

$f'(x) = 1 \cdot \ln(3) \cdot 1 \cdot 3^x$

$f'(x) = \ln(3)\, 3^x$

$answer: f'(x) = \ln(3)\, 3^x$

21) $f(x) = 3^{5x}$

Rule needed $f'(x) = c\ln(a)g'(x)a^{g(x)}$

$c = 1$

$a = 3$

$g(x) = 5x$

$g'(x) = 5$

$f'(x) = 1 \cdot \ln(3) \cdot 5 \cdot 3^{5x}$

answer: $f^{'}(x) = 5\ln(3)\,3^{5x}$

$f'(x) = 5\ln(3)3^{5x}$

#23-38: Find the derivative of each logarithmic function

23) $y = \ln(4x)$

Rule needed $f(x) = cln[g(x)]$ $f'(x) = \frac{cg'(x)}{g(x)}$ $c\ is\ a\ constant$

$c = 1$

$g(x) = 4x$

$g'(x) = 4$

$y' = \frac{1 \cdot 4}{4x}$

answer $y' = \frac{1}{x}$

$y' = \frac{\cancel{4}}{\cancel{4}x}$

$y' = \frac{1}{x}$

25) $y = \ln(8x^2)$

Rule needed
$f(x) = c\ln[g(x)]$
$f'(x) = \frac{cg\prime(x)}{g(x)}$
c $is\ a\ constant$

$C = 1$

$g(x) = 8x^2$

$g'(x) = 16x$

$$\frac{dy}{dx} = \frac{1 \cdot 16x}{8x^2}$$

$answer$: $\frac{dy}{dx} = \frac{2}{x}$

$$\frac{dy}{dx} = \frac{{}^{2}\cancel{16}\cancel{x}}{\cancel{8}\cancel{x}x}$$

$$\frac{dy}{dx} = \frac{2}{x}$$

27) $f(x) = \ln(2x - 3)$

Rule needed $f(x) = c\ln[g(x)]$ $f'(x) = \frac{cg'(x)}{g(x)}$ $c\ is\ a\ constant$

$C = 1$

$g(x) = 2x - 3$

$g'(x) = 2$

$$f'(x) = \frac{1 \cdot 2}{2x-3}$$

$answer$: $f'(x) = \frac{2}{2x-3}$

$$f'(x) = \frac{2}{2x-3}$$

29) $y = 3xln(5x)$

Rule needed $f(x) = cln[g(x)]$ $f'(x) = \frac{cg\prime(x)}{g(x)}$ $c\ is\ a\ constant$ $c=1 \quad g(x)=5x$

$g'(x)=5 \quad \frac{1\cdot 5}{5x}=1/x$

Also need product rule

First factor $3x$	Second Factor $\ln(5x)$
Derivative 3	Derivative $1/x$
cross multiply top down $3x\cdot\frac{1}{x}=3$	*cross multiply bottom up* $3\ln(5x)$

$$y' = 3 + 3\ln(5x)$$

$$= 3(1 + \ln(5x))$$

OR $3(\ln(5x)+1)$

$answer: y' = 3(\ln(5x) + 1)$

31) $f(y) = y^2 \ln(3y)$

Rule needed $f(x) = c\ln[g(x)]$ $f'(x) = \frac{cg'(x)}{g(x)}$ $c \text{ is a constant}$

$C = 1$

$g(y) = 3y$

$g'(y) = 3$

$\frac{1 \cdot 3}{3y} = \frac{3}{3y} = 1/y$

Also need product rule

First factor y^2	Second Factor $\ln(3y)$
Derivative $2y$	Derivative $1/y$
cross multiply top down $y^2 \cdot \frac{1}{y} = y$	*cross multiply bottom up* $2y\ln(3y)$

$f'(y) = y + 2y\ln(3y)$

$f'(y) = y(1 + 2\ln(3y))$

OR

answer $f'(y) = y(2\ln(3y) + 1)$

$f'(y) = y(2\ln(3y) + 1)$

33) $f(x) = log_3(x)$

$f(x) = clog_b[g(x)]$
$f'(x) = \frac{cg\prime(x)}{\ln{(b)}g(x)}$
$c\ is\ a\ constant$
$b > 0$

$c = 1$

$b = 3$

$g(x) = x$

$g'(x) = 1$

$$f'(x) = \frac{1 \cdot 1}{\ln(3)\,x}$$

$$f'(x) = \frac{1}{\ln(3)\,x}$$

$answer$: $f^{'}(x) = \frac{1}{\ln(3)x}$

35) $f(x) = log_3(2x + 7)$

$f(x) = clog_b[g(x)]$
$f'(x) = \frac{cg\prime(x)}{\ln(b)g(x)}$
$c \; is \; a \; constant$
$b > 0$

$C = 1$

$b = 3$

$g(x) = 2x + 7$

$g'(x) = 2$

$$f'(x) = \frac{1 \cdot 2}{Ln(3)(2x+7)}$$

$answer \;\; f'(x) = \frac{2}{\ln(3)(2x+7)}$

$$f'(x) = \frac{2}{Ln(3)(2x+7)}$$

#37-42:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

37) $y = e^{x^2}$

a) Find derivative, then solve derivative equal to zero.

Rule needed for the derivative $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

a) $c = 1$

$g(x) = x^2$

$g'(x) = 2x$

$y' = 1 \cdot 2x \cdot e^{x^2}$

$y' = 2xe^{x^2}$

$2xe^{x^2} = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$

$e^{x^2} = 0$

No Solution

Part a answer $\boxed{x = 0}$

37a) $answer: x = 0$ 37b) $y = 1$

#37-42:

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

39) $y = 3xe^x$

a) Find derivative, then solve derivative equal to zero.

Rule needed for the "e" $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor $3x$	Second Factor e^x
Derivative 3	Derivative e^x
cross multiply top down $3xe^x$	*cross multiply bottom up* $3e^x$

a

$$f'(x) = 3xe^x + 3e^x$$

$$f'(x) = 3e^x(x+1)$$

$$3e^x(x+1) = 0$$

$3e^x = 0$ No Solution

$x+1=0$

$x=-1$

$\boxed{\text{ANSWER PART A} \quad x=-1}$

b) $x = -1$

$y = f(-1) = 3(-1)e^{-1}$

$= -3e^{-1}$

$= -3/e^{1}$

Point $(-1, -3/e)$

Slope $m = 0$

All horizontal lines have slope $m = 0$

39a) $x = -1$ 39b) $y = -3/e$

$y - (-3/e) = 0(x - (-1))$

$y + 3/e = 0$

$-3/e \quad -3/e$

Answer part b $\quad y = -3/e$

a) Find all values of x where the tangent line is horizontal

b) Find the equation of the tangent line to the graph of the function for the values of x found in part a.

41) $y = xe^{2x}$

a) Find derivative, then solve derivative equal to zero.

Rule needed for the "e" $f(x) = ce^{g(x)}$ $f'(x) = cg'(x)e^{g(x)}$ Where "c" is a constant (number without a letter)

Also need the product rule as both factors have an x.

First factor x	Second Factor e^{2x}
Derivative 1	Derivative $2e^{2x}$
cross multiply top down $x \cdot 2e^{2x}$	*cross multiply bottom up* $1e^{2x}$

$$y' = 2xe^{2x} + 1e^{2x}$$

$$y' = e^{2x}(2x+1)$$

$$e^{2x}(2x+1) = 0$$

$e^{2x} = 0$

No Solution

$2x+1=0$

$2x=-1$

$x=-1/2$

$$\boxed{\text{Answer PART A } \quad x = -\frac{1}{2}}$$

b) $x = -\frac{1}{2}$

$y = f(-1/2) = -\frac{1}{2} e^{2 \cdot -1/2}$

$y = -\frac{1}{2} e^{-1}$

$y = -\frac{1}{2} \cdot \frac{1}{e} = \frac{-1}{2e}$

$answer$: 41a) $x = -\frac{1}{2}$ 41b) $y = \frac{-1}{2e}$

Point $(-1/2, -1/2e)$

Slope $M = 0$

$y - (-1/2e) = 0(x - (\frac{-1}{2}))$

$y + \frac{1}{2e} = 0$

$-\frac{1}{2e} \quad -\frac{1}{2e}$

Answer PART b $\boxed{y = -1/2e}$